I. Course Proficiency Purpose:

The purpose of this study guide is to aid the students who wish to take the proficiency assessment for the credit flex option. Items that the student will be required to know for proficiency will be administered in two portions. The first assessment is a written exam with the use of a graphing calculator. The second written exam is used without a graphing calculator.

II. Description of the Assessment Format:

a. The first portion is a written exam consisting of seven (7) free-response questions and thirty-five (35) multiple choice questions to be completed with the aid of a graphing calculator. (120 minute time limit)
b. The second portion is a written exam consisting of eleven (11) free-response and sixty-five (65) multiple choice questions to be completed without the aid of a graphing calculator. (120 minute time limit)

Calculator Policy

The use of a graphing calculator is considered an integral part of the Precalculus course, and is permissible on parts of this Credit Flex exam. Students should use this technology on a regular basis so that they become adept at using their graphing calculators. Students should also have experience with the basic paper-and-pencil techniques of precalculus and be able to apply them when technological tools are unavailable or inappropriate.

Graphing Calculator Capabilities for the Exams

The committee develops exams based on the assumption that all students have access to four basic calculator capabilities used extensively in precalculus. A graphing calculator appropriate for use on the exams is expected to have the built-in capability to:

- Plot the graph of a function within an arbitrary viewing window
- Find the zeros of functions (solve equations numerically)

One or more of these capabilities should provide the sufficient computational tools for successful development of a solution to any exam question that requires the use of a calculator. Care is taken to ensure that the exam questions do not favor students who use graphing calculators with more extensive built-in features.

List of Graphing Calculators

Graphing calculators having the expected built-in capabilities listed above are indicated with an asterisk (*). However, students may bring any calculator on the list to the exam; any model within each series is acceptable.

Casio

FX-6000 series
FX-6200 series
FX-6300 series
FX-6500 series
FX-7000 series
FX-7300 series
FX-7400 series
FX-7500 series
FX-7700 series
FX-7800 series
FX-8000 series
FX-8500 series
FX-8700 series
FX-8800 series
FX-9700 series*
FX-9750 series*
FX-9860 series*
CFX-9800 series*
CFX-9850 series*
CFX-9950 series*
CFX-9970 series*
FX 1.0 series*
Algebra FX 2.0 series *

Hewlett-Packard

HP-9G
HP-28 series*
HP-38G*
HP-39 series*
HP-40 series*
HP-48 series*
HP-49 series*
HP-50 series*

Radio Shack

EC-4033
EC-4034
EC-4037

Sharp

EL-5200
EL-9200 series*
EL-9300 series*
EL-9600 series**
EL-9900 series*

Texas Instruments

TI-73
TI-80
TI-81
TI-82*
TI-83/TI-83 Plus*
TI-83 Plus Silver*
TI-84 Plus*
TI-84 Plus Silver*
TI-85*
TI-86*
TI-89*
TI-89 Titanium*
TI-Nspire*/TI-Nspire CX*
TI-Nspire CAS*/TI-Nspire CX CAS*

Other

Datexx DS-883
Micronta
Smart^2

** The use of the stylus is not permitted.

Note: This list is current as of April 2011; other allowable machines will be added as necessary.

Technology Restrictions on the Exams

You are not permitted to use these items on the Precalculus credit-flex Exams: non-graphing scientific calculators, portable and handheld computers, laptops, electronic writing pads, pocket organizers.

Additionally, you cannot use any graphing calculator models with these features or capabilities: QWERTY (typewriter-like) keypad as part of hardware or software (e.g., TI-92 Plus, Voyage 200); pen-input, stylus or touch-screen (e.g., PalmPilot, personal digital assistant, Casio ClassPad); wireless or Bluetooth capabilities; paper tapes; talk or make noise; require an electrical outlet; have cell phone, audio, or video recording capability; can access the Internet; or camera or scanning capability. Also, the use of hardware peripherals with an approved calculator is prohibited.

Proctors are required to check calculators before the exam. Therefore, it is important for each student to have an approved calculator. Students should be thoroughly familiar with the operation of the calculators they plan to use on the exam. Calculators may not be shared, and communication between calculators is prohibited during the exam. Students may bring to the exam one or two (but no more than two) graphing calculators from the current List of Graphing Calculators.

Calculator memories will be cleared prior to the exam.

Students must not use calculator memories to take test materials out of the room. Students that attempt to remove test materials from the room by any method will have their exam grades invalidated.

Showing Work on the Free-Response Sections of the Exams

Students are expected to show enough of their work for Readers to follow their line of reasoning. To obtain full credit for the solution to a free-response problem, students must communicate their methods and conclusions clearly. Answers should show enough work so that the reasoning process can be followed throughout the solution. This is particularly important for assessing partial credit. Students may also be asked to use complete sentences to explain their methods or the reasonableness of their answers, or to interpret their results.

For results obtained using one of the four required calculator capabilities listed above, students are required to write the setup (e.g., the equation being solved) that leads to the solution, along with the result produced by the calculator. For example, if the student is asked to find the roots of a polynomial, the student is expected to show the equation, the graph and the location on their graph that holds the roots.

When a student is asked to justify an answer, the justification must include mathematical reasons, not merely calculator results. Functions, graphs, tables, or other objects that are used in a justification should be clearly identified.

Exploration Versus Mathematical Solution

A graphing calculator is a powerful tool for exploration, but students must be cautioned that exploration is not a mathematical solution. Exploration with a graphing calculator can lead a student toward an analytical solution, and after a solution is found, a graphing calculator can often be used to check the reasonableness of the solution.

Note: All decimal answers must be correct to three decimal places unless otherwise indicated. Students should be cautioned against rounding values in intermediate steps before a final calculation is made. Students should also be aware that there are limitations inherent in graphing calculator technology; for example, answers obtained by tracing along a graph to find roots or points of intersection might not produce the required accuracy.

III. Proficiency Content:

Modeling & Equation Solving

- Match numerical models with graphical models.
- Describe a numerical model verbally.
- Plot and interpret numerical data.
- Solve equations algebraically and graphically.

Functions & their properties

- Determine if a relation is a function through graphs and formulas.
- Determine the domain and range of a function algebraically and graphically.
- Discover removable and non-removable discontinuities
- Identify local extrema and increasing/decreasing intervals.
- Determine boundedness of an equation.

- Determine whether functions are odd/even/neither.
- Find horizontal and vertical asymptotes of a function.
- Describe end behavior of graphs and functions.

Basic Functions

- Identify each of the 12 basic functions visually and through equation.
  (Identity, Squaring, Cubing, Reciprocal, Square Root, Exponential, Logarithm, Sine, Cosine, Absolute Value, Greatest Integer, Logistic)

- Describe behaviors of the 12 basic functions.

Building Functions from Functions

- Combine functions algebraically.
- Compose functions.
- Find the domain of combined and composed functions.

- Define a function implicitly.

Parametric Relations & Inverses

- Define relations parametrically.
- Find inverses of functions and relations.

Graphical Transformations

- Describe translations algebraically and graphically.
- Describe reflections algebraically and graphically.
- Describe stretches and shrinks algebraically and graphically.

Modeling with Functions

- Identify appropriate basic functions with which to model real world problems.
- Produce specific functions to model data, formulas, graphs, and verbal descriptions.

Linear and Quadratic Functions and Modeling

- Recognize Polynomial functions, stating the degree and leading coefficients.
- Recognize linear and quadratic functions.
- Graph and label linear and quadratic functions.
- Describe the strength and direction of linear correlation.

Power Functions and Modeling

- Identify a power function.
- Write a power function formula.
- Analyze power functions. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End Behavior, Interesting facts).
- Describe formulas in terms of variation and proportion.

Polynomial Functions of Higher Degree with modeling

- Graph polynomial functions
- Predict end behavior of a polynomial function.
- Find real zeros of a polynomial function using graphing and algebraic methods.
- Determine multiplicity of a zero of a polynomial function.
- Use the Intermediate Value Theorem to determine location of real zeros.

Real Zeros of Polynomial Functions

- Divide polynomials using long division.
- Divide polynomials using synthetic division.
- Apply the Remainder Theorem, Factor Theorem, and Rational Zero Theorem.
- Locate lower and upper bounds.

Complex Zeros and the Fundamental Theorem of Algebra

- Factor polynomial with real coefficients.
- Describe a polynomial with complex factors.
- Apply the Fundamental Theorem of Algebra.
- Write a polynomial function as a product of linear factors.
- Apply the Complex Conjugate Theorem.

Graphs of Rational Functions

- Describe graphs of rational functions. (Domain, Range, Continuity, intercepts Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End Behavior, Interesting facts).
- Identify horizontal and vertical asymptotes of rational functions.
- Predict the end behavior of rational functions.
- Describe asymptotic behavior using the concept of limits.

Solving Equations in One Variable

- Solve equations involving fractions using algebra.
- Solve equations involving fractions using graphical technique.
- Identify extraneous solutions.

Solving Inequalities in One Variable

- Use algebra to solve inequalities involving fractions.
- Use the sign chart as an algebraic method of solving inequalities involving fractions.
- Use graphical techniques to solve inequalities involving fractions.

Exponential and Logistic Functions

- Evaluate exponential expressions.
- Identify and graph exponential functions.
- Identify and graph logistic functions.
- Describe and evaluate exponential functions recursively and explicitly.
- Analyze characteristics of exponential and logistic graphs. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End Behavior, Interesting facts).

Exponential and Logistic Modeling

- Use exponential growth, decay and regression to model real life problems.

Logarithmic Functions and their Graphs

- Convert Equations between logarithmic and exponential forms.
- Evaluate common and natural logarithms.
- Graph common and natural logarithms with transformations.

Properties of Logarithmic Functions

- Apply the properties of logarithms to evaluate expressions.
- Apply the properties of logarithms to graph functions. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End Behavior, Interesting facts).
- Apply the properties of logarithms to re-express data.

Equation Solving and Modeling

- Apply the properties of logarithms to solve exponential and logarithmic equations algebraically
- Apply the properties of logarithms to solve application problems.
- Determine orders of magnitude of differing quantities.

Mathematics of Finance

- Use exponential functions and equations to solve business and finance applications related to compound interest and annuities.
- Determine which formulas to use for compound interest non-continuous, compound interest continuous, future value, and present value.

Angles and their Measures

- Convert between radians and degrees.
- Convert in and out of DMS measure
- Calculate arc length
- Read and write bearings an object travels measured clockwise from due north.

Trigonometric Functions of Acute Angles

- Define the 6 trigonometric functions using the lengths of the sides of a right triangle.
- Evaluate Trigonometric functions with a calculator.
- Evaluate Trigonometric functions using the unit circle to obtain exact answers.
- Find the acute angle to satisfy a trigonometric equation using your knowledge of the unit circle.

Trigonometry Extended: The Circular Functions

- Find multiple coterminal angles for any angle measurement.
- Find the reference angle for any angle measurement.
- Determine the sign (+/-) of trigonometric values based on the quadrant.
- Evaluate the 6 trigonometric functions for any given ordered pair.
- Evaluate the 6 trigonometric functions for any angle measurement using its reference angle and quadrant restrictions. (Exact answers for a unit circle angle)

Graphs of Sine and Cosine: Sinusoids

- Describe characteristics of the Sine Function. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End behavior, Interesting facts)
- Describe characteristics of the Cosine Function. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End behavior, Interesting facts)
- Graph Sine and Cosine functions with various transformations
  - Period (Horizontal stretch/shrink)
  - Amplitude (Vertical stretch/shrink)
  - Phase Shift
  - Reflections
  - Vertical Shift

Graphs of Tangent, Cotangent, Secant, and Cosecant

- Describe characteristics of the Tangent Function. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End behavior, Interesting facts)
- Describe characteristics of the Cotangent Function. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End behavior, Interesting facts)
- Describe characteristics of the Cosecant Function. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End behavior, Interesting facts)
- Describe characteristics of the Secant Function. (Domain, Range, Continuity, Increasing/Decreasing, Symmetry, Boundedness, Local extrema, Asymptotes, End behavior, Interesting facts)
- Graph Tangent and Cotangent functions with various transformations
  - Period (Horizontal stretch/shrink)
  - Amplitude (Vertical stretch/shrink)
  - Phase Shift
  - Reflections
  - Vertical Shift
- Graph the Cosecant and Secant functions based on the Sine and Cosine graphs with various transformations
  - Period (Horizontal stretch/shrink)
  - Amplitude (Vertical stretch/shrink)
  - Phase Shift
  - Reflections
  - Vertical Shift

Solving Problems with Trigonometry

- Apply the concepts of trigonometry to solve real world problems

Fundamental Identities

- Use the fundamental identities to simplify trigonometric expressions.
- Use fundamental identities to solve trigonometric equations.

Proving Trigonometric Identities

- Use the fundamental identities to prove equations are identities.

The Law of Sines

- Use the Law of Sines to solve a variety of problems
- Identify when a triangle is the ambiguous case.

The Law of Cosines

- Use the Law of Cosines to solve a variety of problems.
- Use Heron's Formula to find the area of a triangle.

Vectors in the Plane

- Apply the arithmetic of vectors.
- Use vectors to solve real-world problems.
- Discuss the difference between "arrow" and "vectors".
- Describe vectors using Unit Vectors.
- Describe vectors using direction and speed.

Dot Product of Vectors

- Calculate dot products.
- Calculate projections of Vectors.
- Calculate the Angle between two Vectors.
- Prove whether or not two angles are Orthogonal.
- Decompose a Vector into Perpendicular Components.

Parametric Equations and Motion

- Define a parametric equation.
- Graph curves parametrically.
- Solve application problems using parametric equations.
- Eliminate a parameter to discover a corresponding rectangular equation.
- Simulate Motion with a Graphing Utility.

Polar Coordinates

- Convert points from polar to rectangular coordinates.
- Convert points from rectangular to polar coordinates.
- Convert equations from polar to rectangular.
- Convert equations from rectangular to polar.
- Use a Graphing Utility to convert between polar and rectangular coordinates
- Calculate distance using polar coordinates.

Graphs of Polar Equations

- Graph Polar Equations.
- Determine the maximum r-value of a Polar Equation.

- Determine symmetry of Polar Equations.
- Identify and Name special Polar Curves, including Rose, Limacon, Lemniscate, Cardioid, and Spiral of Archimedes.

De Moivre's Theorem and nth Roots

- Represent complex numbers in the complex plane.
- Use the trigonometric form of complex numbers to multiply, divide and raise complex numbers to powers.
- Write complex numbers in trigonometric form.
- Use trigonometric form of complex numbers to simplify some algebraic operations with complex numbers.
- Use De Moivre's Theorem to raise complex numbers to powers.
- Use the trigonometric form of complex numbers to determine all of the nth roots of complex numbers.

Solving Systems of Two Equations

- Solve systems of linear equations graphically.
- Solve systems of linear equations algebraically using substitution and elimination methods.
- Solve non-linear systems using substitution.
- Solve non-linear systems graphically.

Matrix Algebra

- Determine the dimensions of a matrix.
- Find the sum of matrices.
- Find the difference of matrices.
- Find the product of matrices.
- Find the inverse of a matrix.
- Find the scalar product of a matrix.
- Find the determinant of a square matrix.
- Understand the properties of matrices.

Multivariate Linear Systems and Row Operations

- Solve systems of linear equations using Gaussian elimination, the row echelon form of a matrix or inverse matrix methods.
- Determine equivalent systems of linear equations.
- Identify the coefficient matrix of a system of linear equations.
- Use elementary row operations on a matrix.
- Fit a parabolic equation to three-points.

Partial Fractions

- Decompose rational expressions into partial fractions.

1. Writing the decomposition factors
2. denominators with linear factors
3. repeated linear factors
4. irreducible quadratic factors
5. repeated irreducible quadratic factors

Systems of Inequalities in Two Variables

- Solve linear programming problems using graphical methods.
- Solve systems of inequalities using graphical methods.
- Use a graphing utility to graph inequalities

Basic Combinatorics

- Use the multiplication principle of counting to count the number of ways that a task can be done.
- Differentiate between a permutation and a combination scenario.
- Use permutation counting formula to count the number of ways that a task can be done.
- Use combination counting formula to count the number of ways that a task can be done.
- Count the number of subsets of a set with n objects.

The Binomial Theorem

- Expand a power of a binomial using the binomial theorem.
- Expand a power of a binomial using Pascal's Triangle.
- Find the coefficient of a given term of a binomial expansion.
- Create and analyze Pascal's Triangle.
- Discover a recursion formula for Pascal's Triangle.

Probability

- Identify a Sample Space.
- Calculate probabilities in a sample space.
- Calculate conditional probabilities in a sample space with equally likely outcomes.
- Calculate conditional probabilities in a sample space with unequally likely outcomes.
- Use Venn Diagrams and Tree Diagrams to count objects in a sample space.

Sequences

- Express Arithmetic sequences explicitly.

- Express Arithmetic sequences recursively.
- Express Geometric sequences explicitly.
- Express Geometric sequences recursively.
- Find limits of convergent sequences.
- Use a graphing utility to investigate arithmetic and geometric sequences.
- Describe characteristics of the Fibonacci Sequence.

Series

- Use sigma notation to describe a sum of a sequence.
- Find finite sums of terms in an arithmetic sequence.
- Find finite sums of terms in a geometric sequence.
- Find sums of convergent geometric series.

Mathematical Induction

- Use the principal of mathematical induction to prove mathematical generalizations.

Statistics and Data (Graphical)

- Distinguish between categorical and quantitative variables.
- Use various kinds of graphs to display data.
    1. Stemplot
    2. Bar chart
    3. Pie chart
    4. Histogram
    5. Time plot
- Use frequency tables to describe data.

Statistics and Data (Algebraic)

- Use measures of center to describe quantitative data.
- Use the five-number summary to describe quantitative data.
- Use a boxplot to describe quantitative data.
- Use standard deviation and normal distribution to describe quantitative data.

- Discuss the difference between a parameter and statistic.
- Use a frequency table to analyze data.
- Use the 68-95-99.7 (empirical) rule to describe normal distributions.

Parametric and Polar

- ★ Graph a parametric equation. Find maximum values and intersections.
- ★ Convert a parametric equation to a rectangular equation.
- ★ Plot points on the Polar Coordinate System.
- ★ Convert rectangular coordinates to polar coordinates.
- ★ Convert polar coordinates to rectangular coordinates.
- ★ Convert rectangular equations to polar equations.
- ★ Convert polar equations to rectangular equations.

IV. Suggested Resources:

- Precalculus Text: Precalculus Graphical, Numerical, Algebraic 7th Edition; Pearson 2007
- www.coolmath.com
- www.mathwords.com
- www.purplemath.com
- www.math.com
- www.khanacademy.com

Vocabulary

Absolute Extrema

Algebraic Models

Asymptote

Boundedness

Composition of Functions

Continuity

Decreasing Functions

Domain

End Behavior

Function Definition

Function Notation

Functions from data

Graphical Models

Hidden Behavior

Increasing Functions

Inverse Functions

Local Extrema

Numerical Models

Parametric Equations

Parent Functions

Range

Symmetry

Transformations

- Translations (Horizontal/Vertical)
- Reflections
- Horizontal & Vertical Stretches/Shrinks

Vertical Line Test

Zero Factor Property

Average Rate of Change

Complex Conjugate Zeros

End Behavior of polynomial functions

Extraneous Solutions

Polynomial Inequalities

Intermediate Value Theorem

Limits and Asymptotes

Linear Correlation

Long Division of Functions

Monomial Functions

Multiplicity

Polynomial Functions

- Term
- Standard Form
- Coefficients
- Leading Term
- Linear
- Quadratic
- Cubic
- Quartic

Power Function

Rational Functions

Rational Inequalities

Rational Zeros Theorem

Synthetic Division of Functions

Upper and Lower Bounds

Variation

- Direct
- Inverse
- Joint

Zeros of a polynomial functions

Annual Percentage Rate (APR)

Annual Percentage Yield (APY)

Annuities

- Future Value
- Present Value

Common Logarithm

Compounded Interest

Continuously Compounded Interest

Decay Factor

Exponential Decay

Exponential Functions

Exponential Growth

Growth Factor

Half-Life

Limit to Growth

Logarithmic Functions

Logistic Decay

Logistic Functions

Logistic Growth

Natural Logarithm

Properties of Logarithms

- Product Rule
- Quotient Rule
- Power Rule

Transcendental Functions

Arc Length

Bearing

Circular Function

Cosecant

Cosine

Cotangent

Coterminal Angle

Damped Oscillation

Damping Factor

Degrees

Period

Periodic

Periodic Functions

Phase Shift

Radians

Reference angle

Secant

Sine

Sinusoid

Standard Position

Tangent

Unit Circle

Wrapping Function

Heron's Formula

Law of Cosines

Law of Sines

Trigonometric Identities

- Cofunction
- Even/Odd
- Pythagorean
- Quotient
- Reciprocal

Angle between vectors (lines)

- Orthogonal Vectors
- Perpendicular Vectors

Applications of Vectors

Coordinate Conversion

De Moivre's Theorem

Distance; Using Polar Coordinates

Dot Product

Equation Conversion

Lines and Line segments

Multiplication and Division of Complex Numbers

Parametric Equations

- Eliminating the parameter
- Parametric Curves

Polar Coordinates

Polar Curves

- Rose, Limacon, Cardioids, Lemniscate, Spiral of Archimedes

Powers of Complex Numbers

Projecting one vector onto another

Roots of Complex Numbers

Simulating Motion

Special Polar Curves

Symmetry

Trig form of Complex Numbers

Vectors in the plane

- Direction Angles
- Two dimensional vectors
- Unit vectors

- Vector Operations

Work

Gaussian elimination

Matrices

- Addition
- Determinant
- Dimensions
- Identity
- Inverse
- Multiplication
- Subtraction

Multivariate Linear Systems

Partial fraction decomposition

Reduced Row Echelon form

Row Echelon Form

Row Operations

Scalar

Solving Systems

- Elimination method
- Graphical method
- Substitution method

Zero Matrix

Arithmetic Sequence

Binomial Distribution

Binomial Theorem

Boxplot

Categorical Data

Combinations

Conditional Probability

Continuous

Convergence

Deduction

Discrete

Factorial

Finite sequence

Five number summary

Frequency table

Geometric sequence

Histogram

Infinite sequence

Limits of infinite sequences

Mathematical Induction

Mean

Median

Mode

Normal Distribution

Parameter

Pascal's Triangle

Permutations

Sample Space

Sequences

Series

Standard Deviation

Statistic

Stemplot

Summation Notation

The Tower of Hanoi

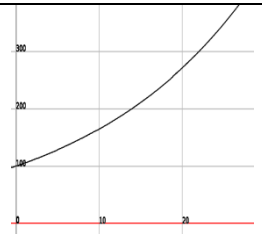
Time plot

Tree diagram

Variance

Venn diagram

Formula Sheet

Special Growth: Compound Interest (non-continuous)	$A(t) = P\left(1+\frac{r}{n}\right)^{n\bullet t}$	$A(t) \rightarrow$ *Amount* at time t $P \rightarrow$ Principal $r \rightarrow$ annual interest rate (decimal) $n \rightarrow$ number of compound periods per year $t \rightarrow$ number of years	$A(x) = 100\left(1+\frac{.05}{4}\right)^{4\bullet x}$ Invest \$100 @ 5% annual interest compounded Quarterly for x years	 Graph shows 25 years of growth
Special Growth: Compound Interest (continuous)	$A(t) = Pe^{r\bullet t}$	$A(t) \rightarrow$ *Amount* at time t $P \rightarrow$ Principal $e \rightarrow$ the natural number e approx. = 2.7182818 $r \rightarrow$ annual interest rate (decimal) $t \rightarrow$ number of years	$A(x) = 100 \bullet e^{.05\bullet x}$ Invest \$100 @ 5% annual interest compounded continuously for x years	 Graph shows 25 years of growth

$$FV = P\frac{(1+i)^n - 1}{i}$$

$$PV = P\frac{1-(1+i)^{-n}}{i}$$

Parabolas with Vertex (h, k)

• **Standard equation**	$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
• **Opens**	Upward or downward	To the right or to the left
• **Focus**	$(h, k+p)$	$(h+p, k)$
• **Directrix**	$y = k-p$	$x = h-p$
• **Axis**	$x = h$	$y = k$
• **Focal length**	p	p
• **Focal width**	$\|4p\|$	$\|4p\|$

See Figure 8.7.

Ellipses with Center (h, k)

• **Standard equation**	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
• **Focal axis**	$y = k$	$x = h$
• **Foci**	$(h \pm c, k)$	$(h, k \pm c)$
• **Vertices**	$(h \pm a, k)$	$(h, k \pm a)$
• **Semimajor axis**	a	a
• **Semiminor axis**	b	b
• **Pythagorean relation**	$a^2 = b^2 + c^2$	$a^2 = b^2 + c^2$

See Figure 8.17.

Hyperbolas with Center (h, k)

• **Standard equation**	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
• **Focal axis**	$y = k$	$x = h$
• **Foci**	$(h \pm c, k)$	$(h, k \pm c)$
• **Vertices**	$(h \pm a, k)$	$(h, k \pm a)$
• **Semitransverse axis**	a	a
• **Semiconjugate axis**	b	b
• **Pythagorean relation**	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$
• **Asymptotes**	$y = \pm\frac{b}{a}(x - h) + k$	$y = \pm\frac{a}{b}(x - h) + k$

See Figure 8.26.